EXPANSION OF A VARIABLE-CONDUCTIVITY PLASMOID WITH SMALL HYDROMAGNETIC-INTERACTION PARAMETER IN AN EXTERNAL MAGNETIC FIELD

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 8, No. 4, pp. 14–20, 1967

The author considers the induction interaction of an expanding variable-conductivity plasmoid which is temperature dependent. The external uniform magnetic field is specified as an arbitrary function of time.

The following two problems are solved assuming a small hydromagnetic-interaction parameter (product of the magnetic Reynolds number and the ratio of the characteristic values of the magnetic pressure to the static plasma pressure): expansion at constant velocity and expansion into free space.

The work A done by the plasma against the electrical body forces and the Joule losses Q in the plasma per unit time are calculated.

A relative analysis of the degree of isoentropicity of the expansion process or the internal efficiency $\eta = (A - Q)/A$ as a function of the magnetic Reynolds number, adiabatic curve, and plasmoid shape (plane-symmetric and axisymmetric) is given.

The results of this calculation can be used as a rough estimate of the efficiency of the MHD method for converting thermal energy into electrical energy.

Consider the one-dimensional plane-symmetric and axisymmetric expansions of an ideal inviscid conducting gas in an external uniform magnetic field. We shall make use of the following assumptions: 1) for the plane-symmetric case, at time $t = 0$ the gas is found within the interval $-a_0 \leq r \leq a_0$ which is bounded along the r-axis but not in the other two directions perpendicular to the r-axis.

For axial symmetry and $t = 0$ we have a cylindrical column of conducting gas which has a radius $r = a_0$ and is infinite along the z-axis; 2) the magnetic field $H(t)$ outside the plasma is known and parallel to the z-axis (perpendicular to the direction of gas motion); 3) the displacement and heat-conduction currents are ignored; 4) as in [1], we assume expansion is uniform and specify the velocity of the medium as $v = ra'(t)/a(t)$, where $a(t)$ is an unknown law governing the motion of the gas boundary; 5) the quantity SR_m (hydromagnetic-interaction parameter) is much less than 1, where S is the ratio of the characteristic magnetic pressure to the static gas pressure and R_m is the magnetic Reynolds number; 6) the gas conductivity σ is a function of temperature and defined by the formula

$$\sigma = \sigma_0 (T / T_0)^n \qquad (n \geqslant 0). \tag{1}$$

Here σ_0 is the conductivity for T_0 chosen as the temperature scale. Expansion takes place in the direction of the r-axis.

Given the above assumptions (1)–(5), the dimensionless nonstationary MHD equations in Lagrange coordinates ξ, τ have the form

$$\frac{\partial H}{\partial \tau} + (\alpha + 1)\frac{a'(\tau)}{a(\tau)} H = \frac{1}{R_m a^2(\tau)} \frac{1}{\xi^\alpha} \frac{\partial}{\partial \xi}\left(\frac{\xi^\alpha}{\sigma(T)} \frac{\partial H}{\partial \xi}\right),$$

$$\left(\xi = \frac{r}{a(\tau)},\ \tau = t\right), \qquad \varkappa M_0^2 \xi a'(\tau) a(\tau) = -\frac{1}{\rho}\frac{\partial P}{\partial \xi},$$

$$\left(\varkappa = \frac{c_p}{c_v},\ M_0 = \frac{v_0}{\sqrt{\varkappa P_0/\rho_0}},\ R_m = 4\pi\sigma_0 a_0 v_0\right),$$

$$\frac{\partial \rho}{\partial \tau} = -\frac{\alpha+1}{a(\tau)} a'(\tau)\rho, \qquad \frac{\partial P}{\partial \tau} = \varkappa \frac{P}{\rho}\frac{\partial \rho}{\partial \tau}, \qquad P = \rho T, \tag{2}$$

where $\varkappa$ is the adiabatic exponent and M_0 is the Mach number. The prime represents differentiation with respect to the variable τ.

In the plane-symmetric case $\alpha = 0$ and in the cylindrical case $\alpha = 1$. The following notation is used for the dimensionless quantities:

$$H = \frac{H^\circ}{H_{00}}, \qquad P = \frac{P^\circ}{P_0}, \qquad \rho = \frac{\rho^\circ}{\rho_0},$$

$$v = \frac{v^\circ}{v_0}, \qquad t = \frac{v_0 t^\circ}{a_0}, \qquad \sigma = \frac{\sigma^\circ}{\sigma_0}.$$

Here the circle superscript denotes dimensional variables and the zero subscript their characteristic values.

In the second and fourth equations of system (2) the second terms on the right which are of order SR_m are omitted (assumption (5)).

The two last equations in (2) are integrated directly:

$$\rho = f(\xi)\, a^{-(\alpha+1)}(\tau), \qquad P = \varphi(\xi)\, \rho^\varkappa. \tag{3}$$

Here $f(\xi)$ and $\varphi(\xi)$ are functions of the Lagrange coordinate ξ; in this case, one can be arbitrarily specified. We assume that $\varphi(\xi)$, which characterizes the initial gas-particle entropy distribution, is such an arbitrary function.

Substituting (3) into the second equation in (2) and using the separation-of-variables method we obtain the following two equations for determining the functions $f(\xi)$ and $a(\tau)$:

$$\frac{d}{d\xi}(\varphi f^\varkappa) = \lambda \varkappa M_0^2 \xi f, \qquad a''(\tau) = -\lambda a^{-[\varkappa + \alpha(\varkappa - 1)]}(\tau). \tag{4}$$

Upon integrating (4) we obtain

$$f(\xi) = \frac{1}{\varphi^{1/\varkappa}(\xi)}\left[\lambda(\varkappa - 1) M_0^2 \int \frac{\xi\, d\xi}{\varphi^{1/\varkappa}(\xi)} + C\right]^{1/\varkappa - 1}, \tag{5}$$

$$\int_1^a \left[C_1 - \frac{2\lambda}{(1-\varkappa)(1+\alpha)} z^{(1-\varkappa)(1+\alpha)}\right]^{-1/2} dz = \tau. \tag{6}$$

Formula (5) defines the gas density distribution with respect to ξ, while (6) characterizes the law governing the motion of the plasma boundary ($\xi = \pm 1$). A solution similar to (6) has been obtained in [2]. In view of the space symmetry of the problem we shall henceforth consider the solution in the region $\xi \in [0, 1]$.

Thus, for the gasdynamic quantities, we have the following expressions which depend on the three arbitrary constants C, C_1, and λ and the arbitrary function $\varphi(\xi)$:

$$v = \xi\left\{C_1 - \frac{2\lambda}{(1-\varkappa)(1+\alpha)}[a(\tau)]^{(1-\varkappa)(1+\alpha)}\right\}^{1/2}, \quad (7)$$

$$\rho = \frac{[\varphi(\xi)]^{-1/\varkappa}}{[a(\tau)]^{1+\alpha}}\left[\lambda(\varkappa-1)M_0^2\int\frac{\xi\,d\xi}{\varphi^{1/\varkappa}} + C\right]^{1/\varkappa-1}, \quad (8)$$

$$P = \frac{1}{[a(\tau)]^{(1+\alpha)\varkappa}}\left[\lambda(\varkappa-1)M_0^2\int\frac{\xi\,d\xi}{\varphi^{1/\varkappa}(\xi)} + C\right]^{\varkappa/\varkappa-1}. \quad (9)$$

Here $a(\tau)$ is defined by relationship (6).

We now consider the solution of the induction equation (first equation in (2)). To do this we separately consider two cases.

a) Expansion with a constant boundary velocity (λ = = 0). We specify the initial and boundary conditions as

$$v(\xi, 0) = \xi, \qquad P(\xi, 0) = 1, \quad (10)$$

$$H(\xi, 0) = b = \begin{cases}0,\\1,\end{cases} \quad \left.\frac{\partial H}{\partial \xi}\right|_{\xi=0} = 0, \; H(1, \tau) = H_0(\tau). \quad (11)$$

The characteristic quantities will be the velocity of the boundary with respect to the plane (axis) of symmetry, and the pressure, density (for ξ = 0) and magnetic field outside the plasma at τ = 0.

Equations (6)–(9), (10), and (11) yield

$$a(\tau) = 1 + \tau, \qquad v = \xi, \qquad \rho = (1+\tau)^{-(1+\alpha)},$$
$$P = (1+\tau)^{-\varkappa(1+\alpha)}. \quad (12)$$

From (1) and (3) for $\varphi(\xi) = 1$ and the equation of state $P = \rho T$ for the dimensionless conductivity it is not difficult to obtain the following relationship as a function of density:

$$\sigma = \rho^{(\varkappa-1)n}. \quad (13)$$

Substituting this expression for σ and expression (12) for $a(\tau)$ into the induction equation in (2) we obtain

$$(1+\tau)^{2-(1+\alpha)(\varkappa-1)n}\frac{\partial H_1}{\partial \tau} = \frac{1}{R_m}\frac{1}{\xi^\alpha}\frac{\partial}{\partial \xi}\left(\xi^\alpha\frac{\partial H_1}{\partial \xi}\right),$$
$$H_1 = (1+\tau)^{1+\alpha}\,H(\xi, \tau).$$

From (11) we have the boundary conditions

$$H_1(\xi, 0) = b, \qquad [\partial H_1/\partial \xi]_{\xi=0} = 0, \quad (14)$$

$$H_1(1, \tau) = (1+\tau)^{1+\alpha}H_0(\tau). \quad (15)$$

To solve problem (14), (15) we can use the integral-transformation method for finite values of ξ [3]; omitting the intermediate steps the final expression is

$$H(\xi, \tau) = H_0(\tau) - \frac{1}{(1+\tau)^{\alpha+1}}\sum_{\gamma=1}^{\infty}C_\gamma \times$$
$$\times\exp\left(-\frac{\lambda_\gamma^2}{R_m}\frac{(1+\tau)^{(1+\alpha)(\varkappa-1)n-1}-1}{(1+\alpha)(\varkappa-1)n-1}\right)\times$$
$$\times\left\{\int_0^\tau\frac{d}{d\tau}[(1+\tau)^{1+\alpha}H_0]\times\right.$$
$$\times\exp\left(\frac{\lambda_\gamma^2}{R_m}\frac{(1+\tau)^{(1+\alpha)(\varkappa-1)n-1}-1}{(1+\alpha)(\varkappa-1)n-1}\right)d\tau +$$
$$\left.+1-b\right\}K(\lambda_\gamma\xi). \quad (16)$$

It is assumed that $(1+\alpha)(\varkappa-1)n - 1 \neq 0$; calculation yields

$$C_\gamma = (-1)^{\gamma-1}\frac{4}{(2\gamma-1)\pi}, \quad K(\lambda_\gamma\xi) = \cos\lambda_\gamma\xi,$$
$$\lambda_\gamma = \frac{(2\gamma-1)\pi}{2} \text{ for } \alpha = 0, \quad (17)$$

$$C_\gamma = \frac{2}{\lambda_\gamma J_1(\lambda_\gamma)}, \quad K(\lambda_\gamma\xi) = J_0(\lambda_\gamma\xi) \text{ for } \alpha = 1. \quad (18)$$

Here J_1 and J_0 are, respectively, first-order and zeroth-order Bessel functions of the real argument and the characteristic numbers λ_γ are found from the equation $J_0(\lambda_\gamma) = 0$.

When $(1+\alpha)(\varkappa-1)n - 1 = 0$, instead of (16) we have

$$H(\xi, \tau) = H_0(\tau) - \frac{1}{(1+\tau)^{(1+\alpha)}}\sum_{\gamma=1}^{\infty}\frac{C_\gamma}{(1+\tau)^{\lambda_\gamma/R_m}}\times$$
$$\times\left\{\int_0^\tau\frac{d}{d\tau}[(1+\tau)^{1+\alpha}H_0](1+\tau)^{\lambda_\gamma/R_m}d\tau + \right.$$
$$\left.+1-b\right\}K(\lambda_\gamma\xi). \quad (19)$$

b) Expansion into free space ($\lambda \neq 0$). Here we consider the subcases in which $v(\xi, 0) = 0$ (zero initial velocity) and $v(\xi, 0) = \xi$ (nonzero initial velocity). In the first subcase the velocity scale is the speed of sound when $\tau = 0$, $\xi = 0$; in the second case the time scale is the initial gas velocity for $\tau = 0$, $\xi = 1$. The characteristic pressure and density are the same as in case (a).

Now, from (7), (9) and the condition $\varphi(\xi) = 1$, we have

$$C = 1, \quad C_1 = \nu + \frac{4}{(\varkappa-1)^2(1+\alpha)M_0^2},$$
$$\lambda = -\frac{2}{(\varkappa-1)M_0^2}. \quad (20)$$

The last condition follows from the expression $P(1, \tau) = 0$

$$\nu = 0 \quad \text{for } v(\xi, 0) = 0,$$
$$\lambda = 1 \quad \text{for } v(\xi, 0) = \xi.$$

Moreover, when $\nu = 0$, we should set $M_0 = 1$.

Thus, allowing for (20), from (7)–(9) we obtain

$$v = \xi\left[\nu + \frac{4}{(\varkappa-1)^2(1+\alpha)M_0^2}\left(1 - a^{(1-\varkappa)(1+\alpha)}\right)\right]^{1/2},$$
$$\rho = (1-\xi^2)^{1/\varkappa-1}\frac{1}{a^{1+\alpha}(\tau)}, \quad P = (1-\xi^2)^{\varkappa/\varkappa-1}\frac{1}{a^{\varkappa(1+\alpha)}}. \quad (21)$$

Here $a = a(\tau)$ is defined by relationship (6); when we allow for (20), this relationship takes the form

$$\int_1^a\left[1 + \frac{4}{(\varkappa-1)^2(1+\alpha)M_0^2}\left(1 - z^{(1-\varkappa)(1+\alpha)}\right)\right]^{-1/2}dz = \tau. \quad (22)$$

We substitute the expression for the conductivity

$$\sigma = (1-\xi^2)^n a^{-(\varkappa-1)(1+\alpha)n} \quad (23)$$

into the induction equation in (2) and change to the new dependent variable $H_1 = a^{1+\alpha} H(\xi, \tau)$; we now have

$$a^{2+(1-\varkappa)(1+\alpha)n} \frac{\partial H_1}{\partial \tau} = \frac{1}{R_m} \frac{1}{\xi^\alpha} \frac{\partial}{\partial \xi}\left[\frac{\xi^\alpha}{(1-\xi^2)^n} \frac{\partial H_1}{\partial \xi}\right]. \quad (24)$$

As before, the boundary and initial conditions are written as

$$[\partial H_1/\partial \xi]_{\xi=0} = 0, \quad H_1(1, \tau) = H_0(\tau) a^{1+\alpha},$$

$$H(\xi, 0) = b = \begin{cases} 0 \\ 1 \end{cases}. \quad (25)$$

Since according to (22) it is not possible to obtain an explicit relationship between a and τ in the general case, it is more convenient to perform differentiation with respect to the variable a rather than τ in (24). The operator

$$\frac{\partial}{\partial \tau} = \left[\nu + \frac{4}{(\varkappa-1)^2(1+\alpha)M_0^2}\left(1 - a^{(1-\varkappa)(1+\alpha)}\right)\right]^{1/2} \frac{\partial}{\partial a}. \quad (26)$$

Therefore, calculation in (24) yields

$$\frac{1}{\xi^\alpha}\frac{\partial}{\partial \xi}\left[\frac{\xi^\alpha}{(1-\xi^2)^n}\frac{\partial H_1}{\partial \xi}\right] - R_m \psi(a) \frac{\partial H_1}{\partial a} = 0,$$

$$\psi(a) = a^{2+(1-\varkappa)(1+\alpha)n} \times$$

$$\times \left[1 + \frac{4}{(\varkappa-1)^2(1+\alpha)M_0^2}\left(1 - a^{(1-\varkappa)(1+\alpha)}\right)\right]^{1/2}. \quad (27)$$

Here conditions (25) take the form

$$[\partial H_1/\partial \xi]_{\xi=0} = 0, \quad H_1(1, a) = a^{1-\alpha} H_0(a),$$

$$H_1(\xi, 1) = b. \quad (28)$$

For n = 0, it is easy to write the solution to problem (27), (28) in an analytic form similar to that of case (a):

$$H(a, \xi) = H_0(a) - \frac{1}{a^{1+\alpha}} \times$$

$$\times \sum_{\gamma=1}^{\infty} C_\gamma \exp\left(-\frac{\lambda_\gamma^2}{R_m}\int_1^a \frac{da}{\psi(a)}\right) \times$$

$$\times \left[\int_1^a \frac{d}{da}\left(a^{1+\alpha} H_0\right) \exp\left(\frac{\lambda_\gamma^2}{R_m}\int_1^c \frac{da}{\psi(a)}\right) da + 1 - b\right] K(\lambda_\gamma \xi). \quad (29)$$

Here, C_γ, $K(\lambda_\gamma \xi)$ and λ_γ have the same meaning as in (a). For n ≠ 0 the problem is numerically solved by the grid method [4] using the EVM-20 computer.

We shall calculate the work A done by the plasma against the electric body forces and the Joule losses Q in a conducting gas. The values of both quantities are given in terms of the unit of height; for $\alpha = 0$, however, they are referred to the width a_0 of the plasma column (layer); then, for the dimensionless quantities A° and Q° we have

$$A^\circ = \frac{A}{q} = a^\alpha(\tau)\left[H_0^2(\tau) v(1, \tau) - \int_0^1 H^2 \frac{\partial}{\partial \xi}(\xi^\alpha v)\, d\xi\right] \quad (30)$$

$$Q^\circ = \frac{Q}{q} = \frac{2}{R_m} a^{\alpha-1} \int_0^1 \left(\frac{\partial H}{\partial \xi}\right)^2 \frac{\xi^\alpha}{\sigma} d\xi \quad \left(q = \frac{H_{00}^2}{4\pi^{1-\alpha}} a_0^\alpha v_0\right). \quad (31)$$

Here A° and Q° are the work and Joule losses per unit time and q is the scale unit.

For expansion with a constant boundary velocity ($\lambda = 0$), Eqs. (30) and (31) yield the following expressions after the substitutions of expressions (12), (13), (16), and (19) for v, σ, H and term by term integration over ξ:

$$A^\circ = (\alpha+1) a^\alpha v(1, \tau) \sum_{\gamma=1}^{\infty}\left[2H_0 f_\gamma(\tau)\int_0^1 K(\lambda_\gamma \xi)\xi^\alpha d\xi - \right.$$

$$\left. - f_\gamma^2(\tau)\int_0^1 K^2(\lambda_\gamma \xi)\xi^\alpha d\xi\right], \quad (32)$$

$$Q^\circ = \frac{2}{R_m}\frac{a^{\alpha-1}}{\sigma(\tau)}\sum_{\gamma=1}^{\infty} f_\gamma^2(\tau)\int_0^1\left(\frac{d}{d\xi}K(\lambda_\gamma \xi)\right)^2 \xi^\alpha d\xi,$$

$$f_\gamma(\tau) = \frac{C_\gamma}{(1+\tau)^{1+\alpha}} \times$$

$$\times \exp\left(-\frac{\lambda_\gamma^2}{R_m}\frac{(1+\tau)^{(1+\alpha)(\varkappa-1)n-1} - 1}{(1+\alpha)(\varkappa-1)n-1}\right) \times$$

$$\times \left\{\int_0^\tau \frac{d}{d\tau}\left[(1+\tau)H_0\right] \times\right.$$

$$\left. \times \exp\left[\frac{\lambda_\gamma^2}{R_m}\frac{(1+\tau)^{(1+\alpha)(\varkappa-1)n-1} - 1}{(1+\alpha)(\varkappa-1)n-1}\right] da + 1 - b\right\}. \quad (33)$$

For $(1+\alpha)(\varkappa-1)n - 1 \neq 0$, if $(1+\alpha)(\varkappa-1) \times n - 1 = 0$ we have

$$f_\gamma(\tau) = \frac{C_\gamma}{(1+\tau)^{\lambda_\gamma/R_m + 1 + \alpha}} \times$$

$$\times \left[\int_0^\tau \frac{d}{d\tau}\left[(1+\tau)^{1+\alpha} H_0\right](1+\tau)^{\lambda_\gamma/R_m} d\tau + 1 - b\right].$$

It is easy to evaluate the integrals with respect to ξ. For $\alpha = 0$, Eq. (17) yields

$$\int_0^1 K(\lambda_\gamma \xi)\, d\xi = (-1)^{\gamma-1}\frac{2}{(2\gamma-1)\pi},$$

$$\int_0^1 K^2(\lambda_\gamma \xi)\, d\xi = \frac{1}{2}, \quad \int_0^1 \left[\frac{d}{d\xi}K(\lambda_\gamma \xi)\right]^2 d\xi = \frac{(2\gamma-1)^2\pi^2}{8}.$$

For $\alpha = 1$ the quantities $K(\lambda_\gamma \xi)$ and C_γ are given by relationships (18); here

$$\int_0^1 K(\lambda_\gamma \xi)\,\xi\, d\xi = \frac{J_1(\lambda_\gamma)}{\lambda_\gamma},$$

$$\int_0^1 K^2(\lambda_\gamma \xi)\,\xi\, d\xi = \frac{1}{2} J_1^2(\lambda_\gamma),$$

$$\int_0^1 \left[\frac{d}{d\xi}K(\lambda_\gamma \xi)\right]^2 \xi\, d\xi = \frac{\lambda_\gamma^2}{2}\left[\frac{2}{\lambda_\gamma}J_1(\lambda_\gamma) - J_0(\lambda_\gamma)\right]^2.$$

We should replace $a(\tau)$, $v(1, \tau)$ and $\sigma(\tau)$ by their expressions in (12) and (13).

For $\lambda \neq 0$ and $n = 0$, the values of A and Q are also given by (32) and (33), respectively; the only difference is that

$$f_\gamma(\tau) = \frac{C_\gamma}{a^{1+\alpha}} \exp\left[-\frac{\lambda_\gamma^2}{R_m}\int_1^a \frac{da}{\psi(a)}\right] \times$$

$$\times \left\{\int_1^a \frac{d}{da}(a^{1+\alpha}H_0) \exp\left[\frac{\lambda_\gamma^2}{R_m}\int_1^a \frac{da}{\psi(a)}\right] da + 1 - b\right\}.$$

Here $v(1, \tau)$, $a(\tau)$, and $\sigma(\tau)$ are found from (21)–(23); in the last expression the value of ρ is taken from (21).

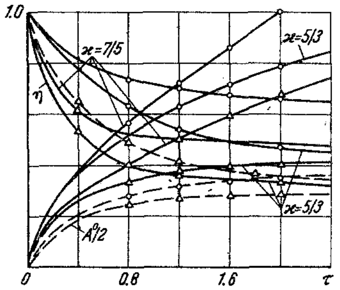

Fig. 1

Numerical calculations were performed for the work A done by the plasma against the electric body forces, Joule losses Q in the plasma per unit time, and the internal efficiency $\eta = (A - Q)/A$ or the degree of isentropicity of the process of plasma expansion in a magnetic field for the case in which the magnetic field at the boundary of the expanding layer (column) of gas is constant ($H_0(\tau) = 1$). The initial distribution of the latter is assumed to be constant and equivalent to the external distribution ($b = 1$).

As an example, Fig. 1 shows some values of A° and η as functions of τ for the case $\lambda = 0$ for $\alpha = 1$, $\varkappa = 5/3$, $7/5$, $R_m = 2$ and 6 (solid lines) and for $\alpha = 0$, $\varkappa = 5/3$, $R_m = 2$ and 6 (dashed lines); for the triangular points in the figure $R_m = 2$, while for the circular points $R_m = 6$.

Here the graph of the coefficient η for $\alpha = 0$, $R_m = 6$, $\varkappa = 5/3$ almost coincides with the graph of the same quantity constructed for $\alpha = 1$, $R_m = 6$, $\varkappa = 7/5$; thus, a graph of the first is not given in the figure.

Figure 2 illustrates A° and η as functions of τ for the case $\lambda \neq 0$ (expansion into free space) for $\alpha = 0$, $\varkappa = 5/3$, $R_m = 2$ and 6. The solid lines correspond to $n = 0$ (constant cross-sectional conductivity $\sigma = \sigma(\tau)$) and the dashed line to $n = 3/2$ (variable cross-sectional conductivity $\sigma = \sigma(\xi, \tau)$).

From the above calculations it follows that:

1) For plane-symmetric expansion ($\alpha = 0$), the values of the coefficient η are greater than for the axisymmetric case ($\alpha = 1$).

2) The value of η depends considerably upon the adiabatic exponent of the gas (i.e., on the working fluid). As is clear from Fig. 1, the coefficient η is greater for a diatomic gas ($\varkappa = 7/5$) than for a monatomic one ($\varkappa = 5/3$).

3) As R_m increases the degree of isentropicity η in the expansion process increases and approaches unity as $R_m \to \infty$.

4) For a given initial magnetic-field distribution $b = 1$, we have $\eta > 0$ for $\tau > 0$, $R_m > 0$ and $\eta = 1$ for $\tau = 0$.

If we assume that at $\tau = 0$ the magnetic field in the plasma is not equal to the external field over the entire cross section (i.e., $b \neq 1$), then for finite R_m there exists some intermediate time $0 \leq \tau \leq \tau_i$, defined by the values of the dimensionless parameters; here $\eta \leq 0$ and only for $\tau > \tau_i$ do we have $\eta > 0$, which was noted earlier in [5, 6].

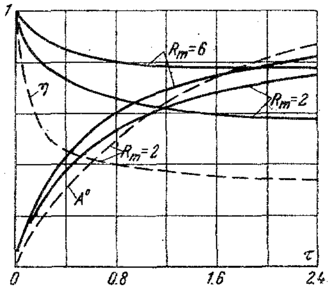

Fig. 2

Figure 3 shows the magnetic-field distribution over the cross section of the conducting plasma layer for the case $\lambda \neq 0$ (expansion into free space) for $\alpha = 0$, $\varkappa = 5/3$, $\eta = 3/2$ and $\nu = 0$. The triangles correspond to $R_m = 2$ and the circles to $R_m = 0.5$.

Note that when the gas has a finite conductivity, part of the work A done by the plasma against the emf forces is dissipated in the working gas in the form of Joule heat. Another fraction of this power goes into changing the magnetic-field energy in the volume occupied by the moving plasma; still another fraction goes into work done against external sources; this is necessary in order to maintain a constant magnetic field ahead of the expanding plasma layer (column).

The author wishes to thank L. A. Zaklyaz'minskii for his valuable comments and his interest in the study.

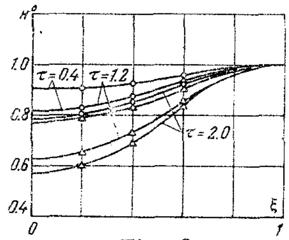

Fig. 3

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12 July 1966 Novosibirsk